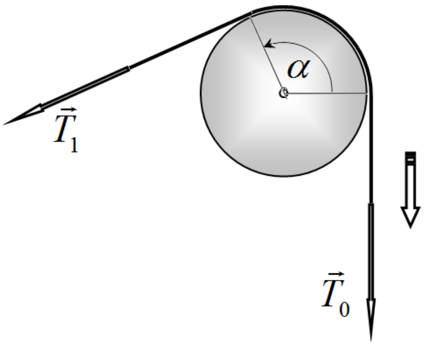
Investigation of the friction force

Part 1. Friction on the cylindrical rod

Theoretical hint:

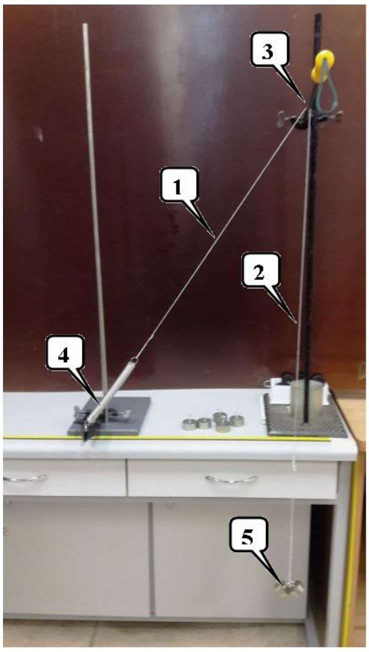


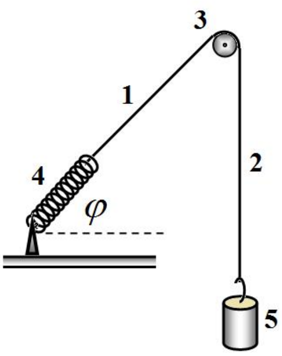
As the thread slides along the side surface of the cylinder, due to the friction, the tension of a rope decreases by a factor of γ. This decrease in tension is described by Euler's formula:

$$T_1 = \gamma T_0 = T_0 \exp(-\mu\alpha) \qquad (1)$$

where μ is a coefficient of friction between the rope and the side surface of the cylinder, α is the angle in which the rope touches the cylinder (subtended angle by the rope).

The goal of this part of the experiment is to check the validity of the Euler's formula, and to determine the coefficient of reduction of tension γ and coefficient of friction μ.





To perform the experimental measurements, the set which is used is shown in the photo and the schematic picture (on the right side). The light and inextensible rope (1-2) is passed over horizontally fixed rod (3). One end of the rope (1) is attached to the spring (4) and the other end of the spring is fixed (4). Loads (5) of different masses are attached to other end of the rope (2). The point at which the spring is attached can be changed, so that one can change the angle φ between the part (1) of the rope with the horizontal.

In all experiments the same spring is used. We denote the spring constant by k, and we can assume that the elastic force of this spring obeys Hook's law

$$F = k\Delta x \, , \qquad (2)$$

where Δx - is the extension of the spring. The spring cannot be compressed. Neglect the mass of the spring.

The measurements technique.

During the experiments the length of the spring x is measured for different masses of the attached loads in the state of rest.

Due to the presence of friction, the weights and spring may be in equilibrium within a certain range of their positions (rest range). To determine the boundaries of this rest range, measurements are carried out as follows:

- the load is lowered down below the equilibrium position, while the spring is stretched, after which the load is allowed to rise very slowly (for this purpose, the rope is held by a hand during its motion) until it stops; thereafter, the length x_1 of the deformed spring is measured;

- the load is lifted above the position of equilibrium, then the loads slowly (the rope is again slightly held by hand) lowered until they stop; after that the length x_2 of the deformed spring is measured .

Measurements of the lengths of the spring are performed by using a metallic ruler. The instrumental error of the measurement is $\delta x = 0{,}5$ mm.
To change the mass of the suspended loads, a different number of standard loads were suspended from the rope, each with a mass equal to $m_{et} = (100{,}0 \pm 0{,}5)g$.

The acceleration due to gravity is assumed to be $g = 9{,}81\frac{m}{s^2}$.

Experiment 1.1

In this experiment rope (1) is located vertically ($\varphi = 90°$).
Results of the measurements of the boundaries of equilibrium positions x_1 and x_2 with different numbers n of suspended standard loads are shown in Table 1.1.

Table 1.1

n	x_1, cm	x_2, cm
1	12,2	10,8
2	16,1	12,4
3	20,2	14,1
4	24,1	16,6
5	27,5	19,2
6	30,7	21,3
7	34,2	23,6

Tasks.

1.1	Obtain theoretical formulas for the dependencies of the lengths of deformed spring $x_1(m)$, $x_2(m)$ on the mass m of suspended loads. Express the results in terms of the following set up parameters: the spring constant k, its length in an undeformed state l_0, the coefficient of the reduction γ of the rope's tension against the rod.

1.2	On one form in the Answer sheet, construct the graphs of the experimental dependencies of the lengths of the spring x_1 and x_2 on the number of suspended loads n .

1.3 Assuming that the obtained experimental dependencies are linear and described by the following functions

$$\begin{aligned} x_1 &= a_1 n + b_1 \\ x_2 &= a_2 n + b_2 \end{aligned}, \qquad (3)$$

Calculate the numerical values of the parameters (a_1, b_1, a_2, b_2) of these dependencies. Estimate the errors of the mentioned parameters.

1.4 Using the obtained data, calculate:
- the length l_0 of the spring in an undeformed state;
- the spring constant k of the spring;
- coefficient of reduction γ of the tension;
- coefficient of friction μ between the rope and the rod;

Estimate the errors of the found values. Give the formulas by which you made your calculations.

Experiments 1.2, 1.3.

In these experiments, the similar measurements have been done for the following positions of the rope (1):
- horizontally $\varphi = 0$ (experiment 2);
- makes an angle of $\varphi = 45°$ with horizontal (experiment 3).

On the Tables 1.2, 1.3 the results of the experiments 2, 3 (dependencies of the lengths of the spring x_1 and x_2 on the number of the suspended standard loads n) are shown.
To simplify your work, the graphs of the obtained dependencies and linear functions which describe these dependencies are given. You can use the values of the coefficients of these functions.

Table 1.2. The rope (1) is horizontally located $\varphi = 0$.

n	x_1, cm	x_2, cm
1	11,8	11,6
2	15,2	13,6
3	18,5	15,9
4	21,5	18,8
5	24,9	21,2
6	28,0	24,0
7	31,6	26,5

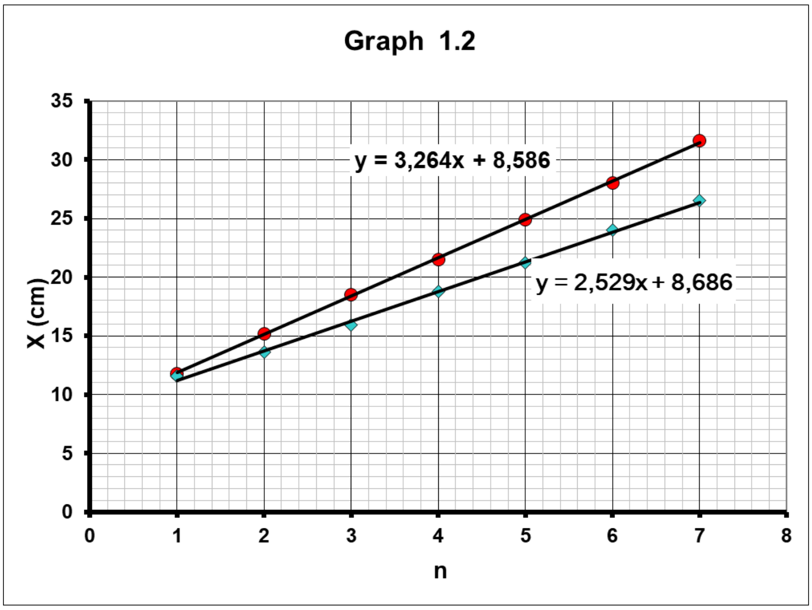
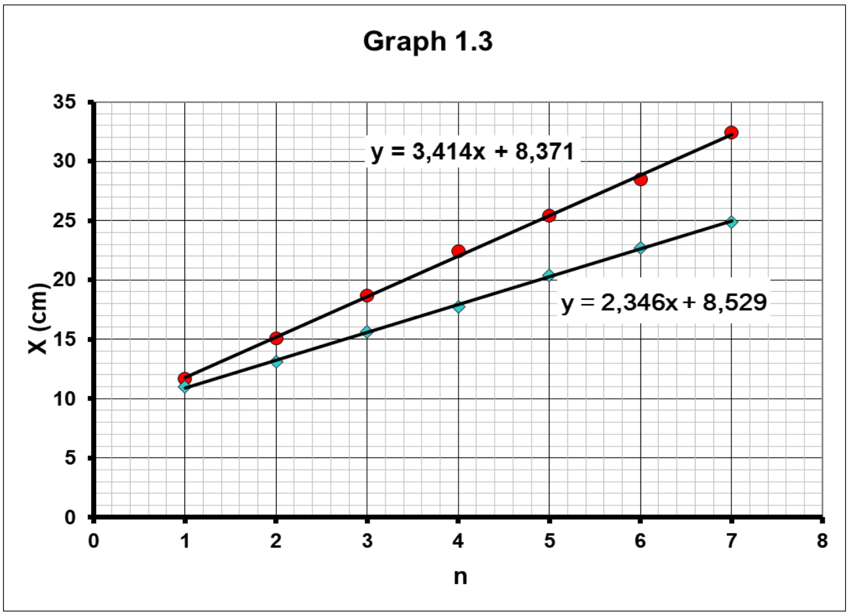


Table 1.3. The rope (1) makes an angle of $\varphi = 45°$.

n	x_1, cm	x_2, cm
1	11,7	11,0
2	15,1	13,1
3	18,7	15,6
4	22,4	17,7
5	25,4	20,4
6	28,5	22,7
7	32,4	24,9



Tasks.

1.5 Using the data provided, calculate values of the coefficients γ for the coverage angles (subtended angles by the rope) in experiments 1.2 and 1.3. Calculate the values of the coefficients of friction μ using the results of experiments 1.2 and 1.3.
Error estimation in this part is not required.

1.6 Determine if Euler's formula (1) is applicable to describe the results of the given data. Give the argument to your answer graphically, or based on additional calculations.

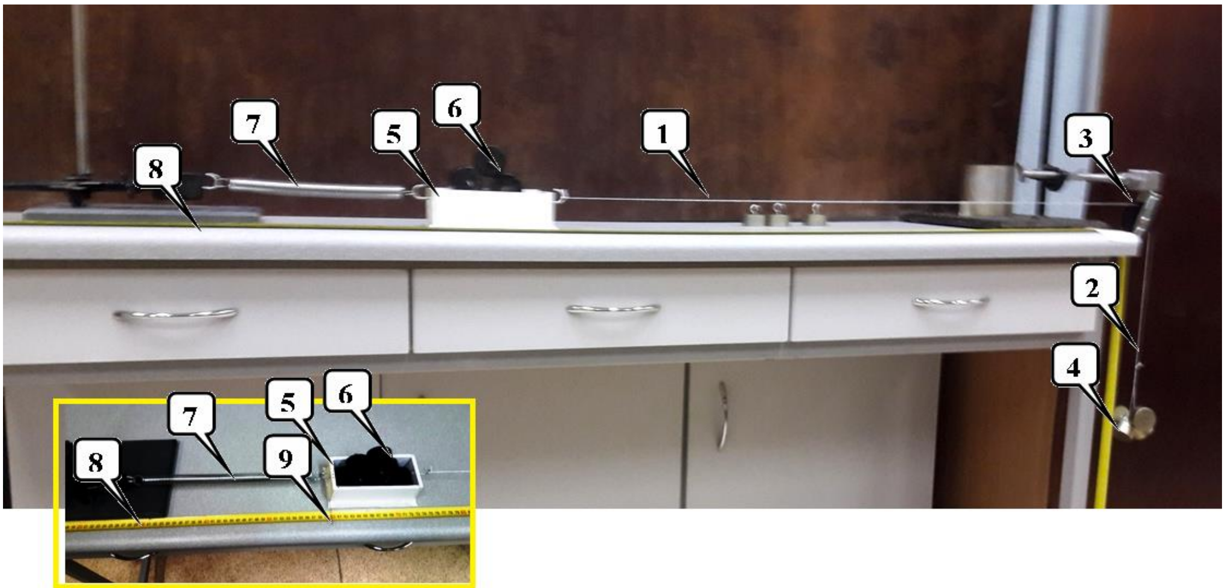
1.7 Using the data of all experiments, specify the value of the coefficient of friction between the rope and the rod. Estimate the error of the specified value of the coefficient of friction. Consider that the errors in calculating the coefficients of friction in all experiments are the same and coincide with the error calculated in the experiment 1.1

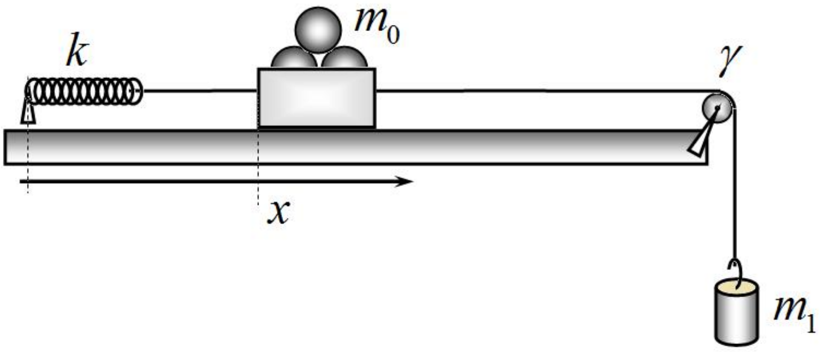
Part 2. The kinetic friction on the horizontal surface.

An estimation of errors in this part of the problem is not required!

In this part of the problem, the friction force between the plastic box and the horizontal surface of the table is investigated.

The following setup is used for measurements.





A strong rope (1-2) is passed over the block (3). Loads (4) are attached to the hanging end of the rope (2). A plastic box (6) is tied to another horizontal section of the rope (1), into which additional weights (6) are placed. A spring (7) is attached to the box, which is also horizontal. The other end of the spring is fixed. A measuring tape (8) is fixed on the surface of the table, with the help of which the coordinate of the edge of the box is measured. To increase the accuracy of coordinate measurements, a thin arrow is attached to the box (9).

We denote the masses of the suspended loads by $m_1 = n_1 m_{et}$, where n_1 is the number of suspended reference loads. We denote the mass of the box with additional weights by $m_0 = n_0 m_{et} + m_b$, where n_0 is the number of additional reference loads placed in the box, m_b is the mass of the box.

The spring constant is denoted by k. Although the same spring is used in this experiment, its spring constant will need to be determined based on the experimental data in this part.

Friction forces act in the axis of the block, which cannot be neglected. The coefficient of weakening of the rope tension due to friction in the axis of the block is denoted by γ. This coefficient is significantly different from the similar coefficient found in Part 1 of this work.

The coordinate of the box is measured from the edge of the table and is not "tied" to this setting. We denote the coordinate of the box by x when the spring is not deformed (you need to determine this value).

Due to the presence of friction (both on the table and in the axis of the block) there is an area of "rest range" within which the box can be at rest.

To measure the boundaries of this region, a technique similar to that used in Part 1 is used. First, the box is displaced to the right, stretching the spring so that the box is out of the rest range zone to the right; after that, the box is released and allowed to slowly move to the left (while holding the thread slightly with the hand) until it stops. In this case, we denote the coordinate of the stopping place by x_1. Then the experiment is repeated in the opposite direction: the box is shifted to the right, taking it out of the rest range zone; then the box is released, and it slowly moves to the right (while the thread is also held by the hand) until it stops. In this case, we denote the coordinate of the stopping place by x_2.

The tables and graphs below show the results of measurements of the dependences of the coordinates of the stop x_1 and x_2 on the number of suspended loads n_1, with a different number of loads n_0 placed in the box. The graphs show the equations of the straight lines

$$\begin{aligned} x_1 &= a_1 n_1 + b_1 \\ x_2 &= a_2 n_1 + b_2 \end{aligned}, \tag{4}$$

describing the obtained linear dependences. Equations are given for linear sections. You can use the parameters of these dependencies in your calculations.

Table 2.1 $n_0 = 2$

n_1	x_2, cm	x_1, cm
1	44,0	47,5
2	45,1	50,6
3	47,8	53,6
4	51,0	56,5
5	53,5	59,6
6	56,1	62,6
7	59,0	65,8

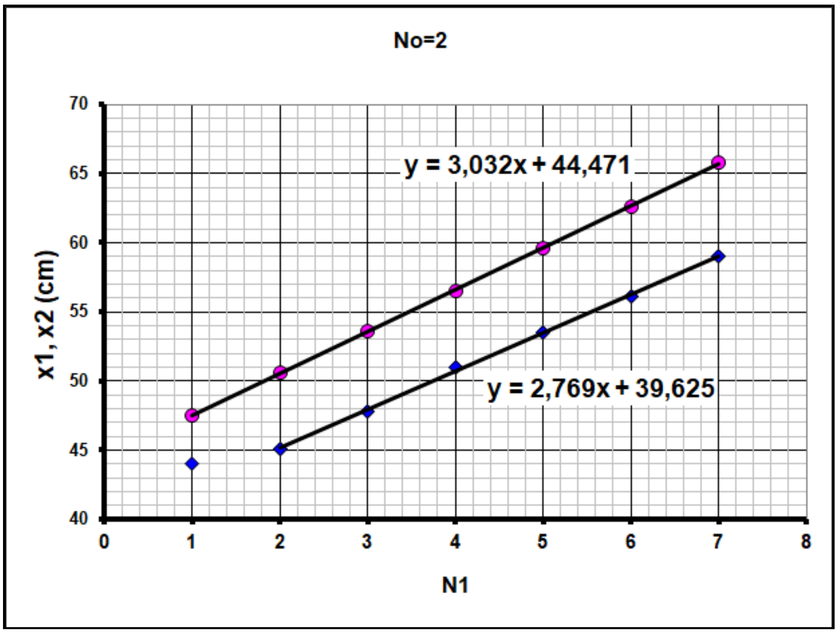


Table 2.2 $n_0 = 3$

n_1	x_2, cm	x_1, cm
1	44,0	48,8
2	44,5	51,8
3	46,7	54,6
4	49,5	57,9
5	52,5	60,5
6	54,8	64,1
7	58,1	66,7

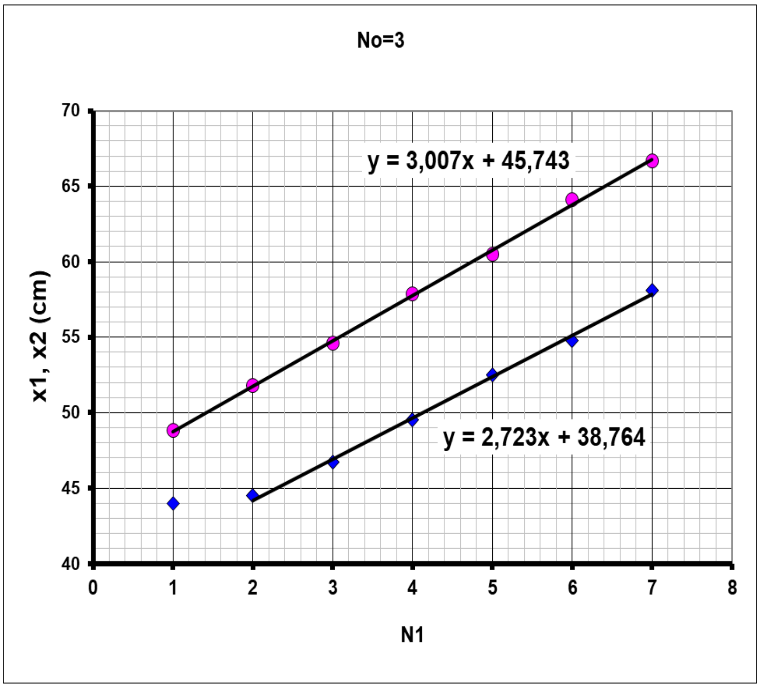


Table 2.3 $n_0 = 4$

n_1	x_2, cm	x_1, cm
1	44,0	49,7
2	44,0	52,5
3	46,0	55,5
4	48,3	58,8
5	51,6	61,9
6	54,1	65,0
7	57,0	67,7

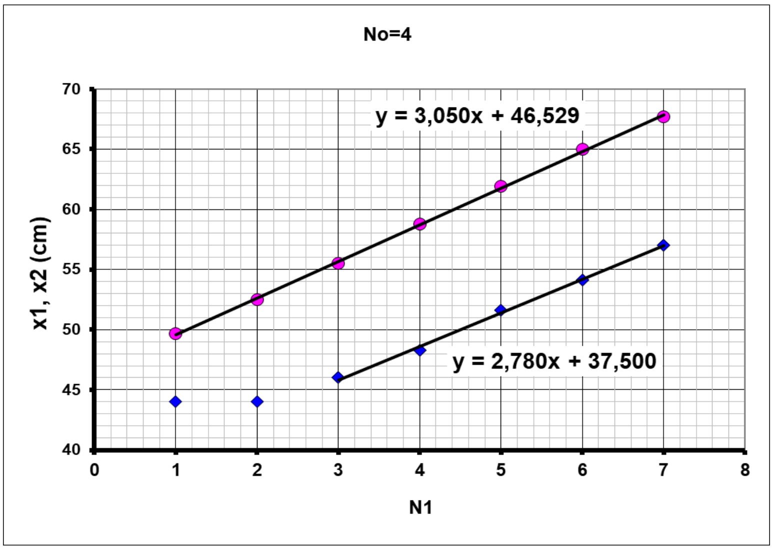


Table 2.4 $n_0 = 5$

n_1	x_2, cm	x_1, cm
1	44,0	51,0
2	44,0	53,0
3	45,1	56,4
4	47,3	59,8
5	50,5	63,0
6	53,3	66,0
7	56,2	68,9

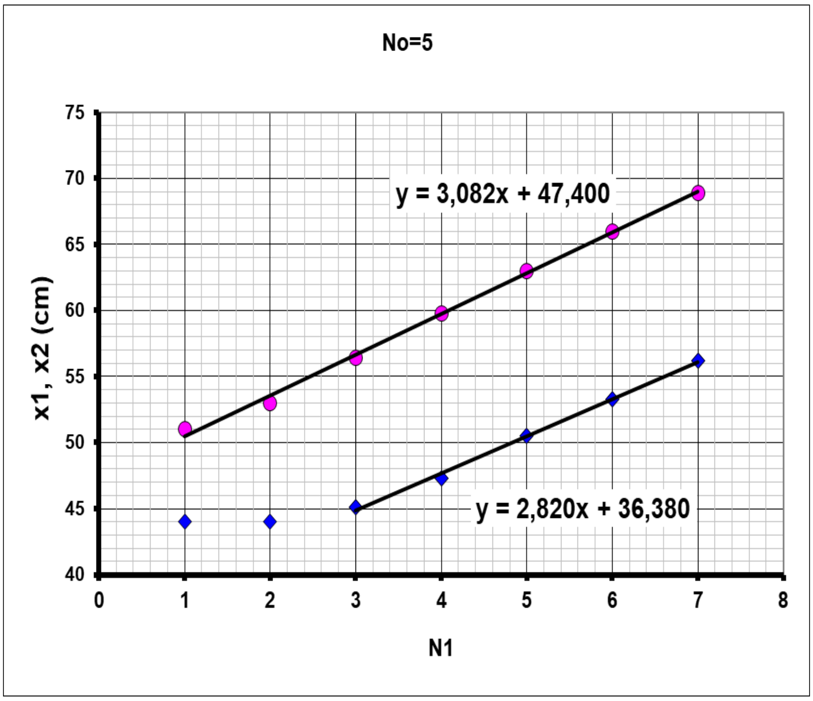
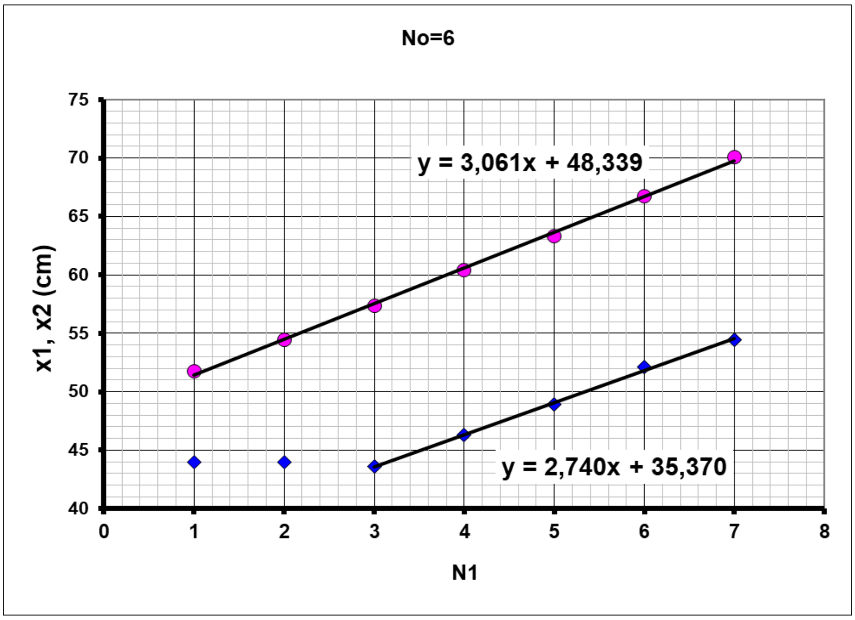


Table 2.5 $n_0 = 6$

n_1	x_2, cm	x_1, cm
1	44,0	51,7
2	44,0	54,4
3	43,6	57,3
4	46,3	60,4
5	48,9	63,3
6	52,1	66,7
7	54,4	70,1



Tasks.

2.1 Determine the coordinate of the box x_0, when the spring is not deformed.

It is not recommended to use the found value of x_0 in this part of the problem, since the error in its determination is large.

2.2 Obtain theoretical formulas describing the dependence of the coordinates x_1 and x_2 of the extreme points of the rest range zone on the mass of the suspended load m_1. Express the results in terms of the spring constant k, the coefficient of reduction of the rope tension on the block γ, and the magnitude of the sliding friction force F.

2.3 For each value of the number of loads n_0 placed in the box, calculate the values of the spring constant k and coefficient γ. Give the formulas for the calculation.

2.4 Using all the results of measurements of this part of the problem, calculate with a minimum error the values of the spring constant $\bar{k}$ and coefficient $\bar{\gamma}$. Give the formulas by which the calculation was carried out.

2.5 Calculate the values of the magnitude of sliding friction force F for each value of the number of loads n_0 in the box. Plot the graph of obtained dependence.

2.6 Using the results of part 2.5, calculate the value of the coefficient of friction of the box on the surface μ and the mass of the box m_b.